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Gauge-Yukawa Unification: Going Beyond GUTs[†]

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Abstract

We discuss the basic idea of the Gauge-Yukawa Unification that is based on the principle of the reduction of couplings. This method of unification relies on the search of successful renormalization group invariant relations among couplings, which do not originate from symmetry principles. The predictive power of Grand Unified Theories can be increased by this method, predicting for instance values of the top quark mass consistent with the recent experimental data. The hope is that this unification attempt might shed further light on the origin of the Yukawa sector of the standard model.

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1 Introduction

The success of the standard model [1, 2] shows that we have at hand a highly nontrivial part of a more fundamental theory of elementary particle physics, which challenges theorists to understand at least some of the plethora of its free parameters.

In constructing realistic field theory models, their renormalizability has played undoubtedly an important rôle [3]. In particular, the structure of the independent parameters in a given theory is basically fixed by its renormalizability. Therefore, the traditional recipe of reducing the number of the independent parameters was to impose symmetries that are compatible with renormalizability. Grand Unified Theories (GUTs) [4, 5] relate in this way not only the gauge couplings of the standard model, but also its Yukawa couplings. In fact, the Georgi-Glashow $SU(5)$ model [4] was very successful in qualitatively predicting the value of the $\sin^2\theta_{\rm W}$ as well as the mass ratio m_τ/m_b [6].

A logical extension of the GUT idea is to attempt to relate the couplings of the gauge and Yukawa sectors, which we would like to call Gauge-Yukawa Unification. However, within the framework of field theory (assuming that all the particles appearing in a theory are elementary), the extended supersymmetry [7] is the only symmetry that could be used to achieve a Gauge-Yukawa Unification (GYU). Unfortunately, theories based on extended supersymmetries seem to introduce more serious and difficult phenomenological problems to be solved than those of the standard model [8].

There exists an alternative way to unify couplings which is based on the fact that within the framework of renormalizable field theory, one can find renormalization group invariant (RGI) relations among parameters and improve in this way the calculability and predictive power of a given theory [9]-[13] (see also ref. [14] for an alternative method). Let us briefly describe this idea below.

Any RGI relation among couplings (which does not depend on the renormalization scale μ explicitly) can be expressed in the implicit form as $\Phi(g_1,\cdots,g_N) ~=~ \text{const.}$, where Φ has to satisfy the partial differential equation (PDE):

$$\mu\,\frac{d\Phi}{d\mu} ~=~ \vec{\nabla}\cdot\vec{\beta} ~=~ \sum_{i=1}^{N}\,\beta_i\,\frac{\partial\Phi}{\partial g_i} ~=~ 0~, \tag{1}$$

where β_i is the β-function of g_i. This PDE is equivalent to the set of the ordinary

differential equations, the so-called reduction equations (REs)[9],

$$\beta_g \frac{dg_i}{dg} = \beta_i \ , \ i = 1, \cdots, A \ , \tag{2}$$

where g and β_g are the primary coupling and its β-function, and the count on i does not include it. Since maximally $(N-1)$ independent RGI relations in the N-dimensional space of couplings can be imposed by Φ_i's, one could in principle express all the couplings in terms of a single coupling g. The strongest requirement is to demand power series solutions to the REs,

$$g_i = \sum_{n=0} r_i^{(n+1)} g^{2n+1} \ , \tag{3}$$

which formally preserve perturbative renormalizability, where $r_i^{(n)}$'s are the expansion coefficients. The possibility of this coupling unification is without any doubt very attractive because a "completely reduced" theory would contain only one independent coupling g. However this ideal case can be unrealistic. Therefore, one often is lead to impose fewer RGI constraints in order to preserve a given theory in a realistic framework, and to introduce the idea of partial reduction [10]. Among the existing possibilities in the framework of supersymmetric $SU(5)$ GUTs, there are two models that are singled out by being strongly motivated [11, 12]. The first one is the $SU(5)$-Finite Unified Theory (FUT) (see refs. [15], and [11] and references therein). In this theory [11] , there exist RGI relations among gauge and Yukawa couplings that yield the vanishing of all β-functions to all orders in perturbation theory [16]. The second is the minimal supersymmetric $SU(5)$ model [17] which can be successfully partially-reduced [12]. The latter is attractive because of its simplicity.

Clearly, in both cases the existence of a covering GUT is assumed so that the unification of the gauge couplings of the standard model is of a group theoretic nature. In ref. [13], we have examined the power of the RGI method by considering theories without covering GUTs, and found that the supersymmetrized model based on the Pati-Salam gauge group is phenomenologically viable. The predictability of the model on the known physics is improved by the present Gauge-Yukawa unification method and, in particular, the model contains only one gauge coupling instead of three.

2 The Principle of the Partial Reduction of Couplings

Here we would like to briefly outline the basic tool of the partial reduction (see refs. [10, 12] for details) which was mentioned above. For many cases, it is convenient to work with the absolute square of g_i's, and therefore we define the tilde couplings by

$$\tilde{\alpha}_i \equiv \frac{\alpha_i}{\alpha} \ , \ i = 1, \cdots, N \ , \tag{4}$$

where $\alpha = |g|^2/4\pi$ and $\alpha_i = |g_i|^2/4\pi$ (i does not include the primary coupling). We assume that their evolution equations take the form

$$\begin{aligned} \frac{d\alpha}{dt} &= -b^{(1)}\,\alpha^2 + \cdots \ , \\ \frac{d\alpha_i}{dt} &= -b_i^{(1)}\,\alpha_i\alpha + \sum_{j,k} b_{i,jk}^{(1)}\,\alpha_j\alpha_k + \cdots \ , \end{aligned} \tag{5}$$

in perturbation theory.

We eliminate t and derive the evolution equations for the tilde couplings

$$\alpha\frac{d\tilde{\alpha}_i}{d\alpha} = (\,-1 + \frac{b_i^{(1)}}{b^{(1)}}\,)\,\tilde{\alpha}_i - \sum_{j,k}\frac{b_{i,jk}^{(1)}}{b^{(1)}}\,\tilde{\alpha}_j\,\tilde{\alpha}_k + \sum_{r=2}(\frac{\alpha}{\pi})^{r-1}\,\tilde{b}_i^{(r)}(\tilde{\alpha}) \ , \tag{6}$$

where $\tilde{b}_i^{(r)}(\tilde{\alpha})$ $(r = 2, \cdots)$ are power series of $\tilde{\alpha}_i$ and can be computed from the r-th loop β-functions. To proceed, we solve the set of the algebraic equations

$$(\,-1 + \frac{b_i^{(1)}}{b^{(1)}}\,)\,\rho_i^{(1)} - \sum_{j,k}\frac{b_{i,jk}^{(1)}}{b^{(1)}}\,\rho_j^{(1)}\,\rho_k^{(1)} = 0 \ , \tag{7}$$

and assume that their solutions $\rho_i^{(1)}$'s have the form

$$\rho_i^{(1)} = 0 \text{ for } i = 1, \cdots, N' \ ; \ \rho_i^{(1)} > 0 \text{ for } i = N'+1, \cdots, N \ . \tag{8}$$

Given the set of the solutions above, we regard $\tilde{\alpha}_i$ with $i \leq N'$ as small perturbations to the undisturbed system which is defined by setting $\tilde{\alpha}_i$, with $i \leq N'$, equal to zero. It is possible [9] to verify, at the one-loop level the existence of the unique power series solution

$$\tilde{\alpha}_i = \rho_i^{(1)} + \sum_{r=2}\rho_i^{(r)}\,(\frac{\alpha}{\pi})^{r-1} \ , \ i = N'+1, \cdots, N \tag{9}$$

of the reduction equations (6) to all orders in the undisturbed system. These are RGI relations among couplings and keep formally the perturbative renormalizability of the undisturbed system. So in the undisturbed system there is only *one independent* coupling, the primary coupling α.

The small perturbations caused by nonvanishing $\tilde{\alpha}_i$, with $i \leq N'$, enter in such a way that the reduced couplings, i.e., $\tilde{\alpha}_i$ with $i > N'$, become functions not only of α but also of $\tilde{\alpha}_i$ with $i \leq N'$. It turned out that, to investigate such partially reduced systems, it is most convenient to work with the partial differential equations

$$
\begin{aligned}
\{ \ \tilde{\beta}\frac{\partial}{\partial\alpha} + \sum_{a=1}^{N'} \tilde{\beta}_a \frac{\partial}{\partial\tilde{\alpha}_a} \ \} \ \tilde{\alpha}_i(\alpha,\tilde{\alpha}) \ &= \ \tilde{\beta}_i(\alpha,\tilde{\alpha}) \ , \\
\tilde{\beta}_{i(a)} \ = \ \frac{\beta_{i(a)}}{\alpha^2} - \frac{\beta}{\alpha^2}\,\tilde{\alpha}_{i(a)} \quad , \quad \tilde{\beta} \ &\equiv \ \frac{\beta}{\alpha} \ ,
\end{aligned}
\tag{10}
$$

which are equivalent to the reduction equations (6) (we let a, b run from 1 to N' and i, j from $N'+1$ to N, in order to avoid confusion). We then look for solutions of the form [10, 12]

$$
\tilde{\alpha}_i \ = \ \rho_i^{(1)} + \sum_{r=1} (\frac{\alpha}{\pi})^{r-1} f_i^{(r)}(\tilde{\alpha}_a) \ , \ i = N'+1,\cdots,N \ ,
\tag{11}
$$

where $f_i^{(r)}(\tilde{\alpha}_a)$ are supposed to be power series of $\tilde{\alpha}_a$. This particular type of solution can be motivated by requiring that, in the limit of vanishing perturbations, we obtain the undisturbed solutions (9) [10, 19], i.e., $f_i^{(1)}(0) = 0$, $f_i^{(r)}(0) = \rho_i$ for $r \geq 2$. Again it is possible to obtain the sufficient conditions for the uniqueness of $f_i^{(r)}$ in terms of the lowest order coefficients. Thus, the partially-reduced system contains the primary coupling α and the disturbing ones $\tilde{\alpha}_a$'s only, thereby increasing the predictive power of the original system.

3 An Example

In the traditional GUT scheme, there exists a covering GUT so that the unification of the gauge couplings of the standard model is of a group theoretic nature. Here we would like to examine the power of the RGI method by considering theories without covering GUTs [13]. Obviously, in order the RGI method for the gauge coupling unification to work, the gauge

couplings should have the same asymptotic behavior. Note that this common behavior is absent in the standard model with three families. A way to achieve a common asymptotic behavior of all the different gauge couplings is to embed $SU(3)_C \times SU(2)_L \times U(1)_Y$ to some non-abelian gauge group, and so we introduce new physics at a very high energy scale and increase the predictability of the model on the known physics by unifying the gauge and part of Yukawa sectors on the basis of the reduction principle. We [13] have found that the minimal phenomenologically viable model is based on the gauge group of Pati and Salam [18]– $\mathcal{G}_{\rm PS} \equiv SU(4) \times SU(2)_R \times SU(2)_L$. We recall that $N=1$ supersymmetric models based on this gauge group have been studied with renewed interest because they could in principle be derived from superstrings [20].

In our supersymmetric, Gauge-Yukawa unified model based on $\mathcal{G}_{\rm PS}$ [13], three generations of quarks and leptons are accommodated by six chiral supermultiplets, three in $(\mathbf{4},\mathbf{2},\mathbf{1})$ and three $(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$, which we denote by $\Psi^{(I)\mu\ i_R}$ and $\overline{\Psi}^{(I) i_L}_{\mu}$ (I runs over the three generations, and μ,ν $(=1,2,3,4)$ are the $SU(4)$ indices while i_R , i_L $(=1,2)$ stand for the $SU(2)_{L,R}$ indices). The Higgs supermultiplets in $(\mathbf{4},\mathbf{2},\mathbf{1})$, $(\overline{\mathbf{4}},\mathbf{2},\mathbf{1})$ and $(\mathbf{15},\mathbf{1},\mathbf{1})$ are denoted by $H^{\mu\ i_R}$, $\overline{H}_{\mu\ i_R}$, and Σ^{μ}_{ν} respectively. They are responsible for the spontaneous symmetry breaking (SSB) of $SU(4)\times SU(2)_R$ down to $SU(3)_C \times U(1)_Y$. The SSB of $U(1)_Y \times SU(2)_L$ is then achieved by the nonzero VEV of $h_{i_R i_L}$ which is in $(\mathbf{1},\mathbf{2},\mathbf{2})$. In addition to these Higgs supermultiplets, we introduce $G^{\mu}_{\nu\ i_R i_L}$ $(\mathbf{15},\mathbf{2},\mathbf{2})$, ϕ $(\mathbf{1},\mathbf{1},\mathbf{1})$ and Σ'^{μ}_{ν} $(\mathbf{15},\mathbf{1},\mathbf{1})$. The $G^{\mu}_{\nu\ i_R i_L}$ is introduced to realize the $SU(4)\times SU(2)_R \times SU(2)_L$ version of the Georgi-Jarlskog type ansatz [21] for the mass matrix of leptons and quarks while ϕ is supposed to mix with the right-handed neutrino supermultiplets at a high energy scale. With these in mind, we write down the superpotential W of the model, which is the sum of the following terms:

$$
\begin{aligned}
W_Y &= \sum_{I,J=1}^{3} g_{IJ}\, \overline{\Psi}^{(I) i_R}_{\mu}\, \Psi^{(J)\mu\ i_L}\ h_{i_R i_L}\ ,\ W_{GJ}\ =\ g_{GJ}\, \overline{\Psi}^{(2) i_R}_{\mu}\, G^{\mu}_{\nu\ i_R j_L}\, \Psi^{(2)\nu\ j_L}\ , \\
W_{NM} &= \sum_{I=1,2,3} g_{I\phi}\ \epsilon_{i_R j_R}\, \overline{\Psi}^{(I) i_R}_{\mu}\ H^{\mu\ j_R}\, \phi\ , \qquad (12) \\
W_{SB} &= g_H\, \overline{H}_{\mu\ i_R}\, \Sigma^{\mu}_{\nu}\, H^{\nu\ i_R} + \frac{g_{\Sigma}}{3}\, \mathrm{Tr}\ [\ \Sigma^3\ ] + \frac{g_{\Sigma'}}{2}\, \mathrm{Tr}\ [\ (\Sigma')^2\, \Sigma\ ]\ , \\
W_{TDS} &= \frac{g_G}{2}\, \epsilon^{i_R j_R} \epsilon^{i_L j_L}\, \mathrm{Tr}\ [\ G_{i_R i_L}\, \Sigma\, G_{j_R j_L}\ ]\ , \\
W_M &= m_h\, h^2 + m_G\, G^2 + m_\phi\, \phi^2 + m_H\, \overline{H}\, H + m_\Sigma\, \Sigma^2 + m_{\Sigma'}\, (\Sigma')^2\ .
\end{aligned}
$$

Although W has the parity, $\phi \to -\phi$ and $\Sigma' \to -\Sigma'$, it is not the most general potential, and, by virtue of the non-renormalization theorem, this does not contradict the philosophy of the coupling unification by the RGI method.

We denote the gauge couplings of $SU(4) \times SU(2)_R \times SU(2)_L$ by α_4 , α_{2R}, and α_{2L} respectively. The gauge coupling for $U(1)_Y$, α_1, normalized in the usual GUT inspired manner, is given by $1/\alpha_1 = 2/5\alpha_4 + 3/5\alpha_{2R}$. In principle, the primary coupling can be any one of the couplings. But it is more convenient to choose a gauge coupling as the primary one because the one-loop β functions for a gauge coupling depends only on its own gauge coupling. For the present model, we use α_{2L} as the primary one. Since the gauge sector for the one-loop β functions is closed, the solutions of the fixed point equations (7) are independent on the Yukawa and Higgs couplings. One easily obtains $\rho_4^{(1)} = 8/9$, $\rho_{2R}^{(1)} = 4/5$, so that the RGI relations (11) at the one-loop level become

$$\tilde{\alpha}_4 = \frac{\alpha_4}{\alpha_{2L}} = \frac{8}{9} \ , \ \tilde{\alpha}_1 = \frac{\alpha_1}{\alpha_{2L}} = \frac{5}{6} \ . \tag{13}$$

The solutions in the Yukawa-Higgs sector strongly depend on the result of the gauge sector. After slightly involved algebraic computations, one finds that most predictive solutions contain at least three vanishing $\rho_i^{(1)}$'s. Out of these solutions, there are two that exhibit the most predictive power and moreover they satisfy the neutrino mass relation $m_{\nu_\tau} > m_{\nu_\mu}$, m_{ν_e}. For the first solution we have $\rho_{1\phi}^{(1)} = \rho_{2\phi}^{(1)} = \rho_{\Sigma}^{(1)} = 0$, while for the second one, $\rho_{1\phi}^{(1)} = \rho_{2\phi}^{(1)} = \rho_{G}^{(1)} = 0$. One then finds that for these two cases the power series solutions (11) take the form

$$\begin{aligned}
\tilde{\alpha}_{GJ} &\simeq \left\{ \begin{array}{l} 1.67 - 0.05\tilde{\alpha}_{1\phi} + 0.004\tilde{\alpha}_{2\phi} - 0.90\tilde{\alpha}_{\Sigma} + \cdots \\ 2.20 - 0.08\tilde{\alpha}_{2\phi} - 0.05\tilde{\alpha}_{G} + \cdots \end{array} \right. , \\
\tilde{\alpha}_{33} &\simeq \left\{ \begin{array}{l} 3.33 + 0.05\tilde{\alpha}_{1\phi} + 0.21\tilde{\alpha}_{2\phi} - 0.02\tilde{\alpha}_{\Sigma} + \cdots \\ 3.40 + 0.05\tilde{\alpha}_{1\phi} - 1.63\tilde{\alpha}_{2\phi} - 0.001\tilde{\alpha}_{G} + \cdots \end{array} \right. , \\
\tilde{\alpha}_{3\phi} &\simeq \left\{ \begin{array}{l} 1.43 - 0.58\tilde{\alpha}_{1\phi} - 1.43\tilde{\alpha}_{2\phi} - 0.03\tilde{\alpha}_{\Sigma} + \cdots \\ 0.88 - 0.48\tilde{\alpha}_{1\phi} + 8.83\tilde{\alpha}_{2\phi} + 0.01\tilde{\alpha}_{G} + \cdots \end{array} \right. , \\
\tilde{\alpha}_{H} &\simeq \left\{ \begin{array}{l} 1.08 - 0.03\tilde{\alpha}_{1\phi} + 0.10\tilde{\alpha}_{2\phi} - 0.07\tilde{\alpha}_{\Sigma} + \cdots \\ 2.51 - 0.04\tilde{\alpha}_{1\phi} - 1.68\tilde{\alpha}_{2\phi} - 0.12\tilde{\alpha}_{G} + \cdots \end{array} \right. ,
\end{aligned} \tag{14}$$

$$
\begin{aligned}
\tilde{\alpha}_{\Sigma} &\simeq \begin{cases} --- \\ 0.40+0.01\tilde{\alpha}_{1\phi}-0.45\tilde{\alpha}_{2\phi}-0.10\tilde{\alpha}_{G}+\cdots \end{cases} , \\
\tilde{\alpha}_{\Sigma'} &\simeq \begin{cases} 4.91-0.001\tilde{\alpha}_{1\phi}-0.03\tilde{\alpha}_{2\phi}-0.46\tilde{\alpha}_{\Sigma}+\cdots \\ 8.30+0.01\tilde{\alpha}_{1\phi}+1.72\tilde{\alpha}_{2\phi}-0.36\tilde{\alpha}_{G}+\cdots \end{cases} , \\
\tilde{\alpha}_{G} &\simeq \begin{cases} 5.59+0.02\tilde{\alpha}_{1\phi}-0.04\tilde{\alpha}_{2\phi}-1.33\tilde{\alpha}_{\Sigma}+\cdots \\ --- \end{cases} .
\end{aligned}
$$

We have assumed that the Yukawa couplings g_{IJ}, except g_{33}, vanish. They can be included into RGI relations as small perturbations, but their numerical effects will be rather small.

So far we have assumed that supersymmetry is unbroken. But we would like to recall that the RGI relations (13) and (14) we have obtained above, remain unaffected by dimensional parameters in mass-independent renormalization schemes. Therefore, in the case of the soft breaking of supersymmetry, these RGI relations are still valid. We then have to translate the RGI relations (13) and (14) into observable quantities. To this end, we apply the renormalization group technique and regard the RGI relations as the boundary conditions holding at the unification scale M_{GUT} in addition to the group theoretic one $\alpha_{33} = \alpha_t = \alpha_b = \alpha_\tau$. One of the large theoretical uncertainties in predicting low energy parameters is the arbitrariness of the superpartner masses. To simplify our numerical analysis we would like to assume a unique threshold M_{SUSY} for all the superpartners. Another arbitrariness is the number of the light Higgs particles that are contained in $h_{i_R i_L}$ and also in $G^{\mu}_{\nu\ i_R i_L}$. The number N_H of the Higgses lighter than M_{SUSY} could vary from one to four while the number of those to be taken into account above M_{SUSY} is fixed at four. In the following, we assume that $N_H = 1$ and examine numerically the evolution of the gauge and Yukawa couplings including the two-loop effects.

In table 1 we present the low energy predictions of the present model for three distinct boundary conditions; $\tilde{\alpha}_{33}(M_{GUT}) = 4.0$, 3.2 and 2.8. All the dimensionless parameters (except $\tan\beta$) are defined in the $\overline{\mathrm{MS}}$ scheme, and all the masses (except for M_{GUT} and M_{SUSY}) are pole masses.

M_{SUSY} [TeV]	$\tilde{\alpha}_{33}(M_{GUT})$	$\alpha_S(M_Z)$	$\alpha(M_{GUT})$	$\tan\beta$	M_{GUT} [GeV]	m_b [GeV]	$m_t[GeV]$
1.6	4.0	0.119	0.046	63.0	0.9×10^{15}	5.01	197.8
1.6	3.2	0.119	0.046	63.0	0.9×10^{15}	4.97	196.1
1.6	2.8	0.119	0.046	63.0	0.9×10^{15}	4.95	195.1

Table 1. The predictions for different boundary conditions, where we have used: $m_\tau = 1.78$ GeV, $\alpha_{em}^{-1}(M_Z) = 127.9$ and $\sin\theta_W(M_Z) = 0.2303$.

Note that the corrections to $\sin^2\theta_W(M_Z)$ that come from a large m_t, i.e., $\sin^2\theta_W(M_Z) = 0.2324 - 10^{-7}\,[138^2 - (m_t/\mathrm{GeV})^2]$, are taken into account above. All the quantities in table 1, except for M_{SUSY}, are predicted; the range of $\tilde{\alpha}_{33}$ is also given by the model (see eq. (14)). We see from table that the low energy predictions are insensitive against the value of $\tilde{\alpha}_{33}$ and moreover that the predicted values are consistent with the recent data of CDF and D0 [22]. Another numerical analysis [13] shows that the present model rather prefers large values of M_{SUSY} ($>$ 400 GeV).

4 Conclusion

The well-known unification attempts [4, 5] assume that all the gauge interactions are unified at a certain energy scale beyond which they are described by a unified gauge theory based on a simple gauge group. The measurements of the gauge couplings at LEP in fact suggest that the minimal supersymmetric $SU(5)$ GUT [17] is very successful when comparing its theoretical values with the experiments. The GUTs can also relate Yukawa couplings among themselves, but the GUT idea alone cannot provide us with the possibility of relating the gauge and Yukawa couplings. In contrast to the GUT scheme, in the alternative unification presented here the symmetry principles do not play a mandatory rôle. The fundamental new concept here is to determine renormalization group invariant relations among couplings of a given GUT which are valid before the breaking of the unifying gauge symmetry. In the particular application discussed here we have found successful GYU yielding predictions which are consistent with the old and recent experimental data. Therefore, it is justified to hope that the present unification

scheme might be able to clarify further the origin of the complex structure of the Yukawa sector of the standard model.

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